CIS 435, Fall 2003 Additional Homework (Due: November 12, 2003)

Solve ALL the problems. Collaboration is prohibited

9 points total

Problem 1. (2 points)
Assume a given integer array $A[1\ldots n]$ is sorted. Write in the pseudocode a function $\text{FIND}(A, n)$ working in $O(n)$ time which finds an element with the largest number of occurrences in $A$.

In case there is more than one element possible the result is the smallest one.

For example if $A = [2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 6]$ then $\text{FIND}(A, 15) = 5$.

Problem 2. (3 points)
Assume $n$ is a positive integer ($n \geq 1$). Write an exact closed formula for $A(n)$ and express $A(n)$ using asymptotic notation $O(f(n))$ (where $f(n)$ is as simple as possible), $A(n)$ is the value computed by the following function ($i \% 3$ denotes here $i \mod 3$)

```c
int A(int n)
{
    int i, s ; s =0 ;
    for(i=1; i<=3*n; i++)
        s = s + (i \% 3) * i ;
    return(s) ; }
```

Problem 3. (2 points)
Let $F_n$ be the $n$-th Fibonacci number:

$$F_0 = 0, \quad F_1 = 1, \quad F_2 = 1, \quad F_3 = 2, \quad F_4 = 3, \quad F_5 = 5, \quad F_6 = 8, \ldots$$

These numbers satisfy: $F_{n+2} = F_{n+1} + F_n$.

Prove by induction that for $n > 0$:

$$F_0 + F_2 + F_4 + F_6 + \ldots F_{2n} = F_{2n+1} - 1.$$

Problem 4. (2 points)
Assume we have the array

$$A = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].$$

How will this array look after performing $\text{Build-Max-Heap}(A)$ using the description of the function $\text{Build-Max-Heap}(A)$ from the textbook.