Problem 1 (2 points)

Suppose $n$ people are arranged in a circle. Beginning with the person 1 we proceed around the circle and remove every second person (not counting removed persons). The last removed element is denoted by $J(n)$. Compute the number $J(2004)$.

Problem 2 (2 points)

Assume you have in the memory an integer table $A[1 \ldots n]$ which represents a MaxHeap. Explain how to find the seventh largest element in $A$ in $O(1)$ time. For example:

$$Seventh([50, 40, 7, 30, 10, 5, 6, 3, 20, 1, 2]) = 6.$$  

Problem 3 (2 points)

Assume we have a set of 11 coins $1, 2, \ldots, 11$, all of them are of the same weight except exactly one fake coin which has a different weight. We have a balance scale with which we can compare any two disjoint sets of coins, the only information about the input we can get is by an operation of weighing, denoted abstractly by $(S_1 <= S_2)$ where $S_1, S_2$ are disjoint subsets of the set of indices $\{1, 2, \ldots, n\}$. The coins with indices in $S_1$ are placed on the left of the scale and with indices in $S_2$ are put on the right side. The output of the operation is $<$, (left branch in the decision tree), $=$ or $>$ (right branch).

For example if the fake coin is 5 and it is lighter then the genuine coin then the operation $(2, 5, 3 <= 1, 6, 9)$ gives the output $<$.  

Draw a ternary decision tree which describes how to find the fake coin with at most 3 operations $<=$ and check if it is lighter or heavier. The output (at each leaf) is an integer, the number of fake coin, together with $+$ (denoting that it is heavier) or $-$. For example the output 7+ means that the fake coin is 7 and it is heavier.

Problem 4 (2 points)

Draw the binary decision tree which uses comparisons "$<" between given 4 distinct unknown integers $a_1, a_2, a_3, a_4$. For example an internal node can have the comparison "$a_3 < a_1$".

The longest top-down branch of the decision tree should have at most 4 comparisons. In the leaves there are answers, the pairs $(i, j)$, where $a_i$ is the maximum and $a_j$ is the minimum element in the sequence.