Problem 1. (3 points)

What is the minimum cost of a Huffman code for 13 items with weights:

\[ 2, 4, 3, 5, 7, 11, 1, 1, 3, 6, 2, 6, 8 \]

Draw a corresponding optimal Huffman tree.

Problem 2. (Red-black trees) (3 points)

Draw red-black tree which results after inserting elements

\[ 14, 13, 12, 11, 1, 10, 9, 8, 7, 6, 5, 4, 3, 2, 15 \]

(in this order) into initially empty red-black tree using algorithm from the class.

Indicate which vertices are black (square nodes) and which are red (round nodes).

Problem 3. (Towers of Hanoi) (2 points)

Write the configuration of the Tower of Hanoi problem for 5 elements after 20 moves. All elements 1, 2, 3, 4, 5 are to be moved from the tower (stack) 1 to the tower 3 using minimal number of moves. Additionally the tower 2 is used. It is not allowed to place larger element above a smaller one. One move consists in shifting top element from one tower to another tower, as its top element. The configuration specifies which elements are on tower 1, tower 2 and tower 3. Elements on one tower are listed in the order top-down, written horizontally with the top on the left.

For example the initial configuration is \((< 12345 >, < \emptyset >, < \emptyset >)\), the final one is \((< \emptyset >, < \emptyset >, < 12345 >)\).

The empty set is denoted here by \(\emptyset\). The problem is also described in "Additional informal lecture notes", see the web page of the course.