Problem 1. (6 points)

Write a closed formula for the sum $S(n)$ of all positive integers $i$ such that $1 \leq i \leq 3 \times n$ and $i$ is not a multiple of 3. For example

$$ S(1) = 1 + 2 = 3, \quad S(2) = 1 + 2 + 4 + 5 = 12. $$

Prove correctness of your formula by induction.

Problem 2. (6 points)

In this question $n$ is a positive integer ($n \geq 1$). Write an exact closed formula for the value of $A(n)$ and express $A(n)$ using asymptotic notation $O(f(n))$, using as simple function $f(n)$ as possible, where $A(n)$ is the following function

```c
int A(int n)
{
    int i,s,j;  s = 0;
    for(i=1; i<=2*n; i++)
        for(j=i; j<=2n-i; j++) s++;
    return(s);
}
```

Problem 3. (5 points)

Suppose $n$ people are arranged in a circle. Beginning with the person 1 we proceed around the circle and remove every second person (not counting removed persons). The last removed element is denoted by $J(n)$.

(a) Compute the numbers $J(100)$ and $J(2^{50} + 50)$. Explain how you were computing it.

(b) What is the formula for $J(2^n + n)$, where $n$ is a positivie integer. Justify your answer.

Problem 4. (5 points)

Draw the decision tree which uses comparisons "<" between given 4 distinct unknown integers $a_1, a_2, a_3, a_4$. The longest top-down branch of the decision tree should have at most 4 comparisons. In the leaves there are answers, the pairs $(i, j)$, where $a_i$ is the largest and $a_j$ is the second largest element in the sequence.

Problem 5. (6 points)

Assume we are given two sorted (in ascending order) arrays $A[1..n]$ and $B[1..n]$ of integers and an integer $x$.

(a) Describe informally a linear time algorithm which returns 1 if $x = a - b$, for some numbers $a \in A$, $b \in B$. If there are no such $a$, $b$ then the value 0 is returned.

(b) Write the algorithm as a function $TEST(n, A, B, x)$ in the pseudo-code.

Problem 6. (6 points)

Assume you have in the memory an integer table $A[1..n]$ which represents a MaxHeap. Explain how to list in sorted order the $\sqrt{n}$ largest elements of $A$ in $O(n)$ time.